

## **Fatigue life analysis of Aluminum Alloy Sheet Under random Vibration**

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**ABSTRACT:** Based on the analysis method of structural vibration fatigue life, the fatigue failure of an aluminum alloy sheet subjected to random vibration load is studied by using finite element analysis software. By using the finite element software of sheet strength and the first six modes are analyzed, which shows that the model meets the engineering requirements; secondly, the vibration fatigue life in frequency domain method Dirlik rain flow distribution model based on the Dirlik formula of fatigue damage and fatigue life of the plates is studied and estimated. The results show that the thin plate satisfies the demand of life under random vibration.

**Keywords:** aluminum alloy sheet; random vibration; finite element analysis; fatigue life estimation

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### **I. INTRODUCTION**

The structure will produce a different vibration response when stimulated by external factors. The vast majority of the structural fatigue failure are related with the vibration, When the vibration frequency is equal to the modal frequency of the structure, it can be regarded as a matter of vibration fatigue. The main vibration fatigue analysis methods of domestic and international were summarized by Minliang Zhou et al [1], who provided technical planning and analysis of vibration fatigue support literature for aircraft design and maintenance. In engineering practice, random vibration is a relatively common form of vibration. Because it is hard to determine the rule of load changing with time in advance, so probability statistics is effective while Time or space coordinate function is intangible. The power spectral density (PSD) function of the structure can be obtained through the analysis of random vibration response. The probability density function, which can be obtained by PSD, can be used to calculate the cumulative fatigue damage and fatigue life of the dangerous parts of the structure by PDF.

In order to estimate the fatigue life of the structure under random vibration environment, Mingzhu Wang [2] proposed a sample method to estimate the fatigue life of random vibration, the sample method can deal with broadband random vibration which is based on the described spectral density in the frequency domain. An Gang [3] discussed the statistical characteristics of random vibration response and the relationship between the dynamic stress of structural response and the conventional fatigue load also finished the response analysis and fatigue life estimation under the action of random vibration load with power spectral density. Fantao Meng [4] analysed the strength characteristics of a wing under random vibration loading by using the method of estimating structural vibration fatigue based on power spectral density function. Roberto Tovo [5] summed up the broadband excitation method to estimate fatigue damage, studied the relationship between the frequency of fatigue damage and different load counting methods, proposed a method of life prediction, which can accurately predict the fatigue life of broadband and narrowband Gauss loads. Minghong Zhou [6-7] studied the frequency domain estimation methods of fatigue life of the broadband random vibration, and verified the applicability and accuracy of various frequency estimation methods. H·Y·Liou [8] proposed a modified model for estimating the fatigue life of structures based on random vibration theory of plastic damage criterion. Cairu Meng [9] discussed two cases of narrow band random load and broadband random load respectively based on power spectral density function of fatigue life estimation of vibration components under random loads. Jiting Zhang etc. [10] proposed the use of random response power spectral density of the characteristic frequency calculated as the average frequency, which is more convenient in the calculation of the amount of fatigue damage. In this paper, we used the Dirlik empirical formula of an aluminum alloy sheet in durable vibration and motor with the Dirlik rain flow distribution model based on frequency domain method of vibration fatigue life to study and estimate the chattering of fatigue damage and fatigue life.

### **1. Structural vibration fatigue analysis theory**

#### **1.1. Structural vibration fatigue theory and its analysis steps**

At present, there is little research on the essential difference between vibration fatigue and general cyclic fatigue. According to the characteristics of load and response, Qihang Yao and Yao Jun [11-13] give the definition of the vibration fatigue: the vibration fatigue is the dynamic response of the structure under the action of the

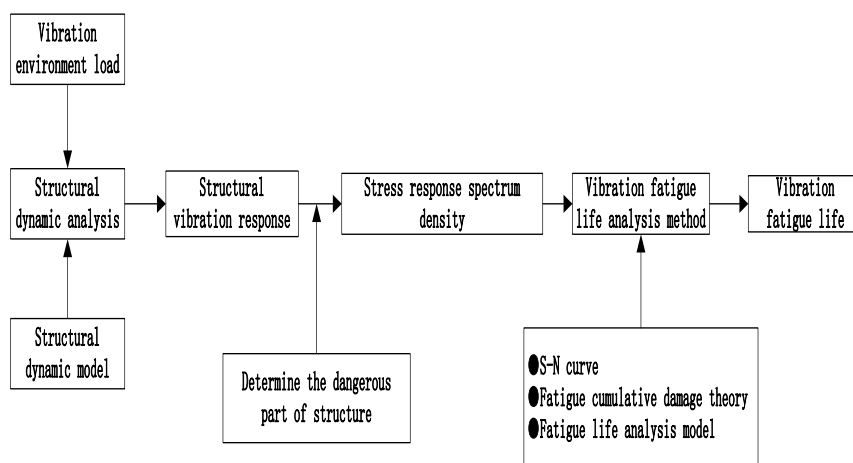
vibration load. Structural vibration fatigue has the following characteristics:

1) In structural vibration fatigue, the load subjected to the structure is often expressed as the dynamic load at or near the loading frequency while the static fatigue load is often very low (e.g. less than 1/2 of the fundamental frequency of the structure).

2) The structural vibration fatigue behavior is closely related to the dynamic characteristics of the structure under load. When we analyzing the structural vibration fatigue characteristics, It is necessary to define the response characteristics due to the structural and dynamic characteristics of the load. At this time, it is necessary to analyze the dynamic characteristics and dynamic response of the whole structure and some parts, so as to clarify the distribution of the response force and the weak parts of the structure while it is difficult to do with the existing static fatigue analysis method .

3) It is also different from the design methods to prevent static fatigue in the design of vibration fatigue and the control of vibration fatigue, the former mainly adopts structural dynamic design and vibration control design technology while the latter mainly carries on the improvement to the bearing characteristic in the structure size, the material and the craft aspect.

The response of vibration fatigue is a random process. Different from general cyclic fatigue, the fatigue life prediction of the structure cannot be predicted by the existing fatigue life estimation method. The dynamic response analysis of structures under vibration load is the first step in the analysis of vibration fatigue life, then estimate the fatigue life of the structure based on the results of dynamic response analysis. The fatigue life analysis of the structure under vibration loading is shown in Figure 1.



**Figure 1** Structural vibration fatigue life analysis procedure

### 1.2 structure frequency domain fatigue life estimation method and Dirlik rain flow distribution model [14]

The structural vibration fatigue life method used in this paper frequency domain analysis method is based on power spectral density. The method for estimating the fatigue life of the structure in frequency domain is to describe the amplitude information of the response in the frequency domain, then to estimate the life in combination with material fatigue life curve and fatigue cumulative damage theory. The method for estimating the fatigue life of structures in frequency domain is based on the spectral parameters of response power spectral density (PSD) in frequency domain, then estimate the fatigue life of the structure according to the fatigue property curve and cumulative damage theory. Frequency domain method has the characteristics of simple thinking and small calculation because of owing the wide attention of the academic and engineering. The peak frequency distribution method is the first method to estimate the fatigue life of the structure, it is based on the peak probability density function proposed by Rice. It was applied for fatigue life estimation for response to broadband random processes, which has a big error in the prediction of structural vibration fatigue life. Amplitude distribution method is the most commonly used method to estimate the fatigue life in frequency domain, it gives the model of rain flow amplitude distribution in response to stochastic processes which is used to estimate Vibration fatigue life. With the development of fatigue research, it is generally believed that the fatigue life estimation is the most suitable for the fatigue damage mechanism. In order to determine the probability density function of stress amplitude  $P(S)$ , Dirlik studied the power spectral density function of 70 different shapes, and proposed an empirical formula of probability density function. The semi empirical formula is composed of an exponential distribution density function and two Rayleigh distribution density functions which can approximate the results

obtained by the rain flow counting method, and apply to any type of power spectrum. In this method, the probability density function of stress amplitude is a function of 4 PSD moments  $M_0, M_1, M_2, M_4$ . The mathematical expression as follows:

$$P(S) = \frac{1}{2\sqrt{m_0}} \left[ \frac{D_1}{Q} e^{-\frac{Z}{Q}} + \frac{ZD_2}{R^2} e^{-\frac{Z^2}{2R^2}} + ZD_3 e^{-\frac{Z^2}{2}} \right] \quad (1)$$

Among them :  $m_0 = \int_0^m f^m G(f) df$ ,  $G(f)$  is a unilateral PSD value at a frequency.

$$D_1 = \frac{2(x_m - \gamma^2)}{1 + \gamma^2}, D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R}$$

$$D_3 = 1 - D_1 - D_2, Z = \frac{S}{2\sqrt{m_0}}, E(P) = \frac{\sqrt{m_4}}{\sqrt{m_2}}$$

$$R = \frac{\gamma - x_m - D_1}{1 - \gamma - D_1 + D_1^2}, \gamma = \frac{m_2}{\sqrt{m_0 m_4}}$$

$$x_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}}, Q = \frac{1.25(\gamma - D_3 - D_2 R)}{D_1}$$

The structural damage is obtained:

$$D = \frac{T\sqrt{m_4}}{2C\sqrt{m_0 m_2}} \int_0^m S^m \left[ \frac{D_1}{Q} e^{-\frac{Z}{Q}} + \frac{ZD_2}{R^2} e^{-\frac{Z^2}{2R^2}} + ZD_3 e^{-\frac{Z^2}{2}} \right] ds \quad (2)$$

In general, it is considered that when the  $D=1$  is damaged, the fatigue life of the structure can be obtained:

$$T = \frac{2C\sqrt{m_0 m_2}}{\sqrt{m_4}} \int_0^m S^m \left[ \frac{D_1}{Q} e^{-\frac{Z}{Q}} + \frac{ZD_2}{R^2} e^{-\frac{Z^2}{2R^2}} + ZD_3 e^{-\frac{Z^2}{2}} \right] ds \quad (3)$$

### 1.3 S-N curve

Fatigue life curve of material is the basis of fatigue life estimation. The curve is a segment of the relationship between the failure life and the stress amplitude under the condition of controlling stress. However, it is not enough to get the S-N curve of the material. The S-N curve of the material only establishes the corresponding relationship between the stress level and the conditional fatigue limit, but not the micro fatigue mechanism [15]. In terms of the fatigue life analysis of concrete structures, the real value is the S-N curve of the structure. There are many factors that affect the S-N curve, such as the dispersion coefficient, the surface processing coefficient, the size factor, etc.. It is difficult to find a suitable S-N curve for the life estimation, so it is necessary to modify the existing S-N curve, so that we can get the curve we need without the test of the structure. This is a common practice for life expectancy.

### 1.4 Miner Cumulative damage theory

There are a lot of theories of fatigue cumulative damage, but the Miner linear cumulative damage theory is widely accepted by theoretical research and engineering practice, especially when dealing with random fatigue problems. Miner Standard think: It is concluded that the ultimate value of the energy absorbed by the specimen is the cause of the fatigue failure, and there is a direct relationship between the energy absorbed by the specimen and the number of load cycles under the condition of a constant amplitude cyclic loading [16]. That is

$$W_i / W = n_i / N \quad (4)$$

In the formula:  $n_i$  is the cyclic number of a constant amplitude load;  $W_i$  is the energy absorbed by the specimen when the number of load cycles is  $n_i$ ;  $N$  is the number of cycles to be downloaded before the failure of the specimen, and the  $W$  is the energy limit before the specimen is destroyed. In this way, the loading process of the specimen before failure is composed of  $n$  different stress levels such as  $\sigma_1, \sigma_2, \sigma_3$  and so on, The life of each stress level is  $N_1, N_2, \dots, N_n$ , Circulation under stress Respectively is  $n_1, n_2, \dots, n_n$ . Then there is the establishment of

$$\sum_{i=1}^r \frac{n_i}{N_i} = 1 \quad (5)$$

the formula is a basic expression of Palmgren-Miner linear cumulative damage criterion.

One of the disadvantages of Miner is that it does not take into account the influence of loading sequence on the fatigue life. In view of this, it is proposed that the relative Miner rule of loading sequence and part shape is considered

$$\sum_{i=1}^r \frac{n_i}{N_i} = D_f \quad (6)$$

In the formula:  $D_f$  is similar parts under similar load spectrum damage and test values.

The formula (5) and formula (6) describe the situation of the discrete stress amplitude, but according to the cumulative idea, the cumulative damage expression can be introduced under the condition of continuous change of the stress amplitude

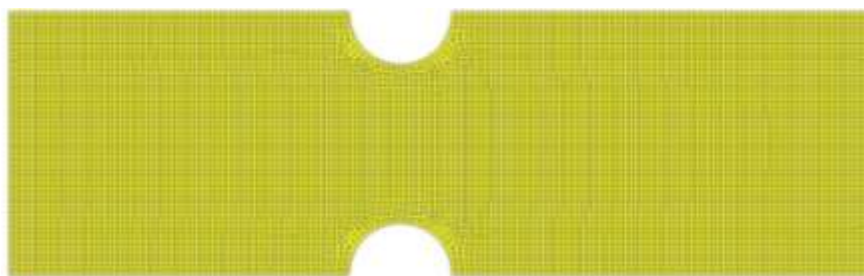
$$D = \int_0^{+\infty} \frac{n(s)}{N(s)} ds \quad (7)$$

In the formula:  $n(s)$  and  $N(s)$  are respectively that the actual number of cycles and the number of broken cycles are continuous function of  $s$  when the stress amplitude is  $s$ .

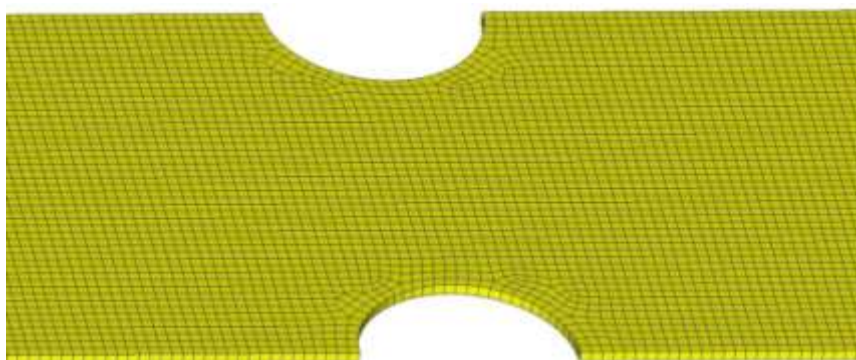
**2. Fatigue life analysis of an aluminum alloy sheet**

**2.1 Finite element model and modal analysis of thin plate**

In order to avoid the modal shape of the high symmetry square model, using rectangular thin plate model. A thin plate model is created in CATIA, the size is 500 \* 150 \* 3mm, the ratio of length to width is 10:3, slotted in the middle, boundary conditions for the left end clamped, As shown in figure 2



(a)

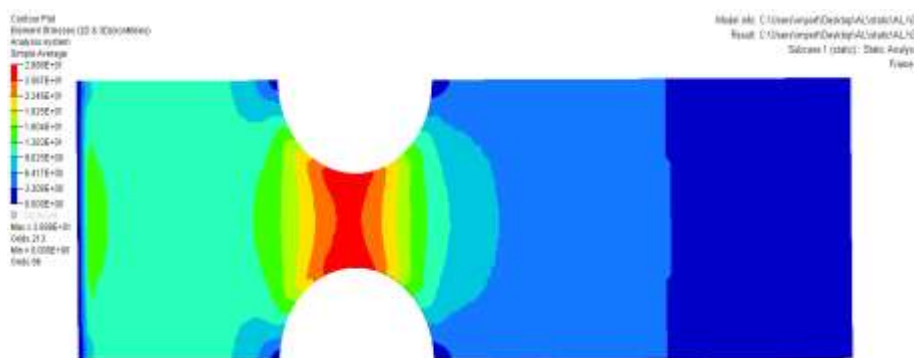


(b)

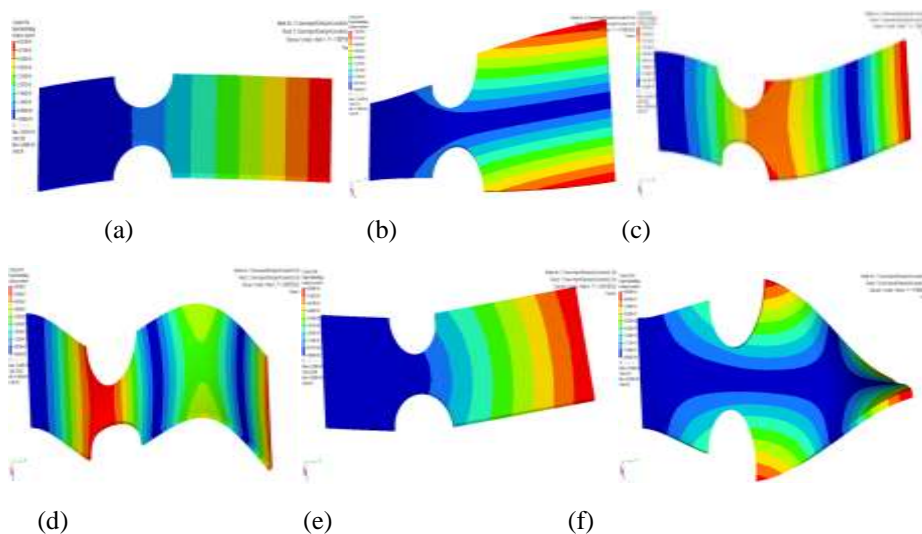
**Figure 2 Finite element model of aluminum alloy sheet**

(a) Whole model (b) Local model

The material is 6061-T651 aluminum alloy, the conventional mechanical properties are:  $E=68.9 \text{ GPa}$ ,  $2800 \text{ kg} / \text{m}^3$ ;  $\sigma_u = 228 \text{ MPa}$ ,  $\nu = 0.33$ . with the help of Hyperworks to calculate strength and the first six natural frequencies (Table 1), the corresponding strength and vibration mode as shown in Figure 3-4



**Figure 3 Strength analysis of aluminum alloy sheet**



(a)

(b)

(c)

(d)

(e)

(f)

**Figure 3 Modal analysis of aluminum alloy sheet**

(a) First order mode (b) second order mode (c) third order mode

(d) Fourth order mode (e) fifth order mode (f) Sixth order mode

**Table 1** The first 6 natural frequencies of thin plate model

Modal order	1	2	3	4	5	6
f/Hz	17.09	107.69	122.84	216.01	348.87	479.70

**2.2 Properties of aluminum alloy**

The aluminum alloy material is 6061-T651, its physical and mechanical properties are as follows: the strength limit is 228MPa; the yield limit is 55.2 MPa; the elastic modulus is 68.9 GPa; the density is  $2800kg / m^3$ ; the Poisson's ratio is 0.33; the density is  $g/cm^3$ .

**2.3 S-N curve analysis of aluminum alloy**

The material of aluminum alloy is 6061-T651, the fatigue curve of 6061-T651 is shown in Figure 6. The S-N curve of the thin plate is calculated by linear fitting and parameter correction. Seeing figure 4.

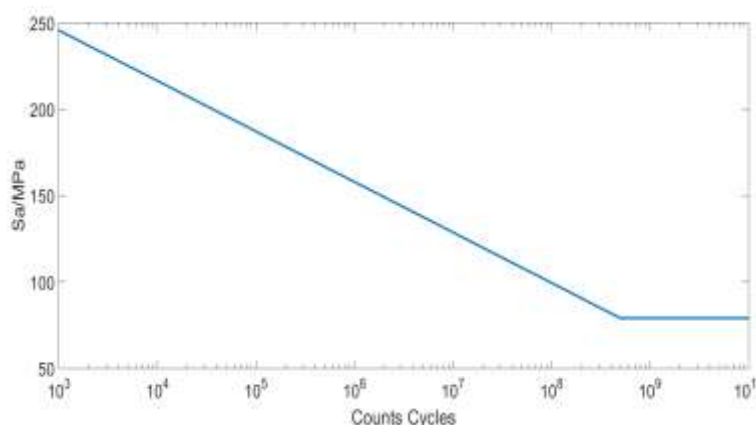


Figure 4 S-N curve

**2.4 Formula derivation**

For the Dirlik formula [17], combined with the S-N curve in the form of power function, we can obtain the expression of the fatigue damage  $\bar{D}$  in time T under the Dirlik formula

$$\bar{D} = \frac{\nu_p}{C} \sigma^\beta \left[ D_1 Q^\beta \Gamma(1 + \beta) + (\sqrt{2})^\beta \Gamma\left(1 + \frac{\beta}{2}\right) (D_2 |R|)^\beta + D_3 \right] \quad (8)$$

In the formula: Function  $\Gamma$  is defined as

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du \quad (9)$$

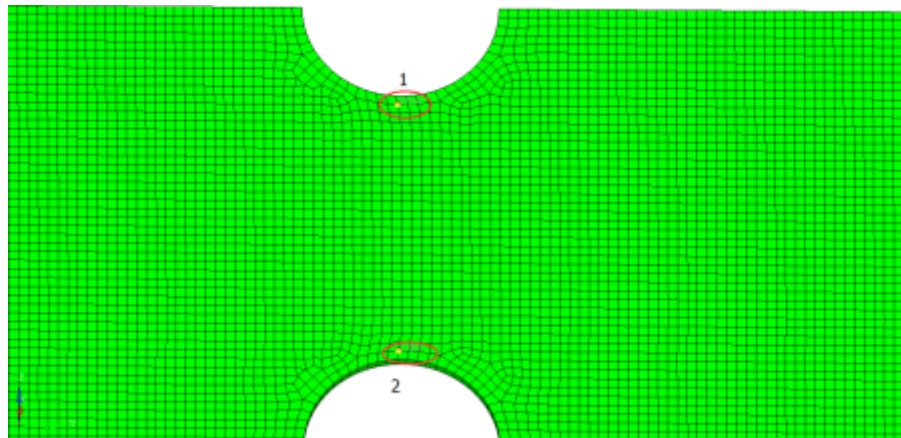
$\nu_p = \frac{1}{2\pi} \sqrt{\frac{m_4}{m_2}}$ ,  $\sigma = \sqrt{m_0}$ ,  $\beta$  and C are Material parameters in the curve. The other parameters are the ones

in the formula!

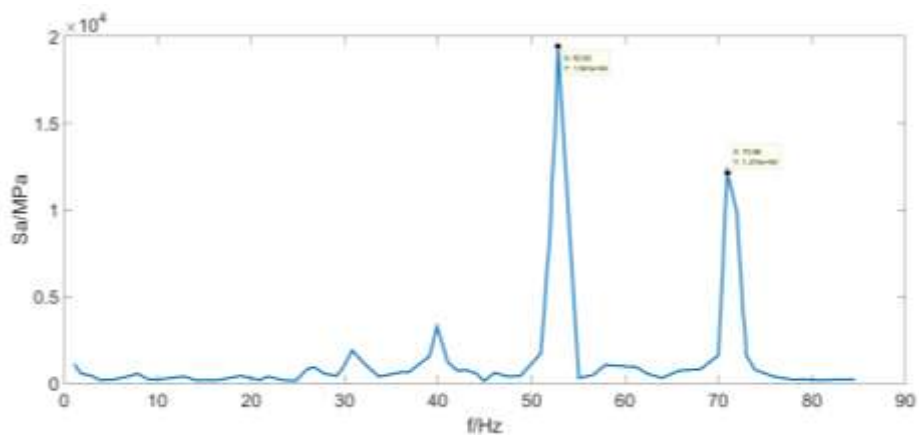
The relevant calculation program is not repeated here!

### 2.5 Fatigue analysis results

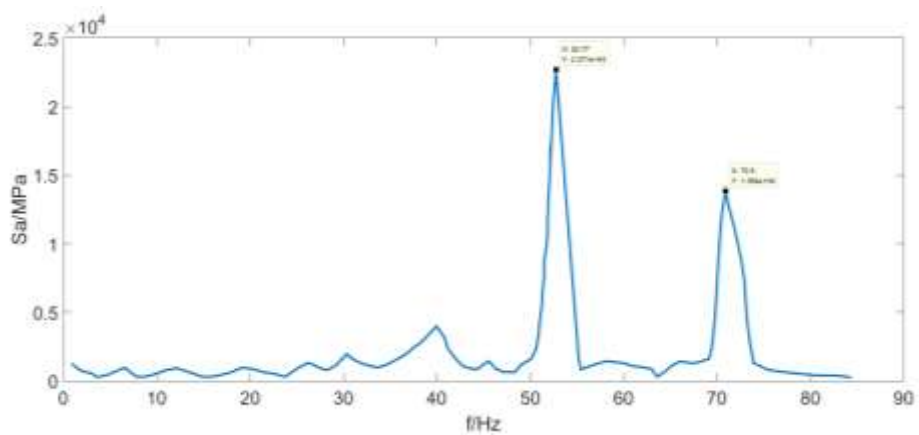
Based on the Dirlik rain flow distribution model of vibration fatigue life in frequency domain method, This paper estimates the fatigue life of aluminum alloy sheet through the derivation of Drilkeexperience life formula and programming to estimate Aluminum Alloy sheet, and the results were compared and analyzed. The power spectrum density of dangerous parts as shown in the figure 5



(a)



(b)



(c)

Figure 5 The power spectrum density of dangerous parts  
(a) Node position (b) Node 17856 PSD (c) Node 13234 PSD

In the table 2, the results of fatigue damage and life calculation of two joints of thin plate are given. It can be seen that the two nodes of the thin plate can meet the requirements of flight life. The life of the node 17856 is the shortest, and the stress response is the largest, and the node 13234 is second, the relative life is longer.

Table 2 Fatigue damage and life calculation results of each node

Position	Node	fatigue life
Dangerous part	17856	$7.53 \times 10^8$
	13234	$2.65 \times 10^8$

### III. CONCLUSION

- (1) At the same time, the two dangerous parts meet the demand of aluminum alloy sheet, The life of the node 17856 is the shortest, and the stress response is the largest, and the node 13234 is second, the relative life is longer.
- (2) It is worthy of attention to designers in the stress response of the larger parts.

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